

$$[2] (\csc x)' = \underline{-\csc x \cot x} \textcircled{\frac{1}{2}}$$

$$(-\csc x \cot x)' = \underline{(\csc x \cot x) \cot x - \csc x (-\csc^2 x)} \textcircled{2}$$

$$= \underline{\csc x \cot^2 x + \csc^3 x} \textcircled{\frac{1}{2}}$$

$$(\csc x \cot^2 x + \csc^3 x)' = \underline{(-\csc x \cot x) \cot^2 x + \csc x (2 \cot x) (-\csc^2 x) + 3 \csc^2 x (-\csc x \cot x)} \textcircled{3}$$

$$= -\csc x \cot^3 x - 2 \csc^3 x \cot x - 3 \csc^3 x \cot x$$

$$= \underline{-\csc x \cot^3 x - 5 \csc^3 x \cot x} \textcircled{\frac{1}{2}}$$

$n$	$f^{(n)}(x)$	$f^{(n)}\left(\frac{5\pi}{4}\right)$
0	$\csc x$	$-\sqrt{2}$
1	$-\csc x \cot x$	$-(-\sqrt{2})(1) = \sqrt{2}$
2	$\csc x \cot^2 x + \csc^3 x$	$(-\sqrt{2})(1)^2 + (-\sqrt{2})^3 = -\sqrt{2} - 2\sqrt{2} = \underline{-3\sqrt{2}} \textcircled{\frac{1}{2}}$
3	$-\csc x \cot^3 x - 5 \csc^3 x \cot x$	$-(-\sqrt{2})(1)^3 - 5(-\sqrt{2})^3(1) = \sqrt{2} + 10\sqrt{2} = \underline{11\sqrt{2}} \textcircled{\frac{1}{2}}$

$$-\sqrt{2} + \frac{\sqrt{2}}{1!} \left(x - \frac{5\pi}{4}\right) - \frac{3\sqrt{2}}{2!} \left(x - \frac{5\pi}{4}\right)^2 + \frac{11\sqrt{2}}{3!} \left(x - \frac{5\pi}{4}\right)^3$$

$$= \underline{\underline{-\sqrt{2}} \textcircled{\frac{1}{2}} + \underline{\underline{\sqrt{2}} \left(x - \frac{5\pi}{4}\right) \textcircled{\frac{1}{2}}} - \underline{\underline{\frac{3\sqrt{2}}{2} \left(x - \frac{5\pi}{4}\right)^2} \textcircled{1}} + \underline{\underline{\frac{11\sqrt{2}}{6} \left(x - \frac{5\pi}{4}\right)^3} \textcircled{1}}}$$

$$[3][a] 1 + e^{-x^2} = 1 + \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = 1 + 1 - \frac{x^2}{1!} + \frac{x^4}{2!} - \frac{x^6}{3!} + \dots$$

$$= \boxed{2 - x^2 + \frac{1}{2}x^4 - \frac{1}{6}x^6} + \dots$$

$$\frac{1 + e^{-x^2}}{\cos x} = \frac{2 - x^2 + \frac{1}{2}x^4 - \frac{1}{6}x^6 + \dots}{1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \dots} \quad \left(\frac{1}{2}\right)$$

$$\left(\frac{1}{2}\right) \left| 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 \right| \sqrt{\frac{2 + \frac{5}{12}x^4 + \frac{2}{45}x^6}{2 - x^2 + \frac{1}{2}x^4 - \frac{1}{6}x^6}}$$

$$\left(\frac{1}{2}\right) \left| 2 - x^2 + \frac{1}{2}x^4 - \frac{1}{360}x^6 \right|$$

$$\frac{5}{12}x^4 - \frac{59}{360}x^6 \leftarrow -\frac{60}{360} - \left(-\frac{1}{360}\right)$$

$$\left(i\right) \left| \frac{5}{12}x^4 - \frac{5}{24}x^6 \right|$$

$$\left(\frac{1}{2}\right) \left| \frac{2}{45}x^6 \right| \leftarrow -\frac{59}{360} - \left(-\frac{5}{24} \cdot \frac{15}{15}\right)$$

$$= -\frac{59}{360} + \frac{75}{360}$$

$$= \frac{16}{360}$$

$$\left(\frac{1}{2}\right) \quad \left(\frac{1}{2}\right) \quad \left(\frac{1}{2}\right)$$

$$\boxed{2 + \frac{5}{12}x^4 + \frac{2}{45}x^6}$$

$$[6] \quad \frac{f^{(6)}(0)}{6!} x^6 = \frac{2}{45} x^6 \quad \left(\frac{1}{2}\right)$$

$$f^{(6)}(0) = \frac{2 \cdot \overset{2}{6} \cdot \cancel{5} \cdot 4 \cdot \cancel{3} \cdot 2 \cdot 1}{\underset{\cancel{9}}{45} \cdot \underset{\cancel{3}}{2}} = 2 \cdot 2 \cdot 4 \cdot 2 = \frac{32}{\left(\frac{1}{2}\right)}$$

$$[4][a] f(x) = \left| \frac{d}{dx} x^2 \arctan 2x^3 \right| \textcircled{1}$$

$$= \frac{d}{dx} x^2 \sum_{n=0}^{\infty} \frac{(-1)^n (2x^3)^{2n+1}}{2n+1}$$

$$= \frac{d}{dx} x^2 \left| \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{6n+3}}{2n+1} \right| \textcircled{2}$$

$$= \frac{d}{dx} \left| \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{6n+5}}{2n+1} \right| \textcircled{\frac{1}{2}}$$

$$= \sum_{n=0}^{\infty} \frac{d}{dx} \frac{(-1)^n 2^{2n+1} x^{6n+5}}{2n+1}$$

$$= \left| \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} (6n+5) x^{6n+4}}{2n+1} \right| \textcircled{1}$$

[b] RADIUS OF CONVERGENCE  $-1 < 2x^3 < 1$   $\left(\frac{1}{2}\right)$

$-\frac{1}{2} < x^3 < \frac{1}{2}$

$-2^{-\frac{1}{3}} = -\frac{1}{\sqrt[3]{2}} < x < \frac{1}{\sqrt[3]{2}} = 2^{-\frac{1}{3}}$   $\left(\frac{1}{2}\right)$

$x = 2^{-\frac{1}{3}} : \left| \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} (6n+5) 2^{-2n-\frac{4}{3}}}{2n+1} \right|$   $\left(\frac{1}{2}\right)$

$= \left| \sum_{n=0}^{\infty} (-1)^n \frac{6n+5}{\sqrt[3]{2}(2n+1)} \right|$   $\left(\frac{1}{2}\right)$

$\left(\frac{1}{2}\right) \lim_{n \rightarrow \infty} \left| (-1)^n \frac{6n+5}{\sqrt[3]{2}(2n+1)} \right| = \lim_{n \rightarrow \infty} \frac{6 + \frac{5}{n}}{\sqrt[3]{2}(2 + \frac{1}{n})} = \frac{6}{2\sqrt[3]{2}} \neq 0$   $\left(\frac{1}{2}\right)$

so  $\lim_{n \rightarrow \infty} (-1)^n \frac{6n+5}{\sqrt[3]{2}(2n+1)} \neq 0$   $\left(\frac{1}{2}\right)$

so  $\sum_{n=0}^{\infty} (-1)^n \frac{6n+5}{\sqrt[3]{2}(2n+1)}$  DIV (DIVERGENCE)  $\left(\frac{1}{2}\right)$

$x = -2^{-\frac{1}{3}} : \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} (6n+5) (-2^{-\frac{1}{3}})^{6n+4}}{2n+1}$

$6n+4 = 2(3n+2)$  IS EVEN

$\left(\frac{1}{2}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} (6n+5) (-1)^{6n+4} 2^{-2n-\frac{4}{3}}}{2n+1}$

$I_0 C = \left(-\frac{1}{\sqrt[3]{2}}, \frac{1}{\sqrt[3]{2}}\right)$   $\left(\frac{1}{2}\right)$

$= \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} (6n+5) 2^{-2n-\frac{4}{3}}}{2n+1}$   $\left(\frac{1}{2}\right)$  DIV (SAME AS ABOVE)